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The Cohesion of Education and Happiness: An Efficiency Assessment of G7 Countries Using Integrated Slack-Based Data Envelopment Analysis and ROC Analysis

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Abstract

The purposes of this study are to assess the efficiency of the cohesion of education and happiness of G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States), and to test the diagnostic performance of input and output variables in determining the efficiency status of G7 countries. For these purposes, the study has a two-stage analysis design. In the first stage; Input Orientation Slack-Based Data Envelopment Analysis (DEA) model was employed using five input variables (PISA Reading Performance, PISA Mathematics Performance and PISA Science Performance) and one output variable (Average Happiness Score). In the second stage, the ROC Analysis was conducted for the diagnostic performance of input and output variables in determining the Efficiency Status of G7 countries. The dataset belongs to 2025 or the nearest year, and it is gathered from OECD Data and the World Happiness Report 2025. While the first stage of analysis design was carried out using the deaR package in the R project, the second stage was carried out using Inonu University Faculty of Medicine, Department of Biostatistics and Medical Informatics, Diagnostic Tests and ROC Analysis Software. According to the first stage of analysis design, it was determined that 5 out of 7 G7 countries are efficient (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States), while the remaining two countries (Japan and the United Kingdom) were found to be inefficient. Besides, all five efficient G7 countries rank first, while Japan ranks last among the seven G7 countries. According to the second stage of analysis design, it was determined that the i2: PISA Mathematics Performance and i3: PISA Science Performance input variables could distinguish the Efficiency Status with the cutoff points (481.901) and (499.542), respectively. As the cohesion of education and happiness efficiency of G7 countries remains largely unexplored in the existing literature, this study stands out by incorporating the Slack-Based Measure and ROC Analysis to fill this research gap.

Keywords: Data Envelopment Analysis, ROC Analysis, Education, G7 Countries

Introduction

OECD countries have made considerable efforts to meet the UN Sustainable Development Goals (SDGs) in areas such as the economy, environment, health and education (Lin et al., 2025). SDG-4 ensures inclusive and equitable quality education and promotes opportunities for lifelong learning for all, whereas SDG-3 guarantees healthy lives and promotes well-being for all at all ages. (United Nations, 2025).

Since healthy people are better able to learn and work, education is more efficient when people are in good physical and mental health. (Lin et al., 2025). Happiness is a social construct that includes variables related to confidence, life satisfaction, and subjective well-being (Mariano et al., 2015). The happiness score is an all-encompassing measure of society's quality (See & Yen, 2018). There is little, inconsistent, and poorly understood data on the relationship between education and happiness, and reports of a negative correlation are frequent (Kristoffersen, 2018).

Happiness is viewed as a remedy for impending crises (Kim, 2024). Happiness as an educational goal has held a paradoxical and unique position, and it continues to be a significant topic in scholarly discussion and educational practice (Kallová, 2021). For social welfare and economic development, education and health outcomes—both psychological and physical—are vital (Verhoeven et al., 2007). It is well known that education significantly improves both individuals' and societies' welfare (Kristoffersen, 2018).

Since everyone wants to be as happy as possible, researching the reasons behind human happiness has been a major focus of fields like philosophy, sociology, psychology, and education (Cordero et al., 2017). In cross-national research, happiness is regarded as one of the common themes, and researchers are examining the connection between happiness and personal traits (Debnath & Shankar, 2014).

According to relevant research, striving for ongoing improvements in social inclusion and education quality can enhance the cycle of productivity, economic growth, and prosperity outcomes. It can also have implications for increased social cohesion (Krstić Srejović et al., 2024). Since efficient educational institutions are those that can maximise their outputs by making the best use of their inputs, new advances in positive psychology and education research have increased interest in happiness (Bhutoria & Aljabri, 2022; Dziechciarz & Szczeciński, 2025).

Data Envelopment Analysis (DEA) offers important insights into resource usage and productivity by comparing the input-output combinations of various decision-making units (DMUs) against the frontier representing best production practices (Rede et al., 2026).

An organisation's decision-making accuracy should be evaluated by contrasting it with the option that would maximise its benefits (Wang, 2014a). In addition to being widely used in medical decision-making, Receiver Operating Characteristic (ROC) graphs—a curve that plots a diagnostic test's sensitivity versus its false positive rate across all potential threshold values for defining positivity—have become more prevalent in business optimisation analysis, health policy making, clinical studies, and health economics in recent years (Fawcett, 2006; Kampfrath & Levinson, 2013; Wang, 2014b). It is a tool for evaluating an instrument's performance (e.g. G. as a component of a measurement system used for classification in an entity test where "true positive rates" (TPR, sensitivity) are plotted against decision risks like "false positive rates" (FPR, fall-out) (Pendrill et al., 2023).

In this context, the study aims to assess the efficiency of the cohesion of education and happiness of G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States), and to test the diagnostic performance of input and output variables in determining the efficiency status of G7 countries. For these purposes, the study has a two-stage analysis design. In the first stage; Input Orientation Slack-Based Data Envelopment Analysis (DEA) model was employed using five input variables (PISA Reading Performance, PISA Mathematics Performance and PISA Science Performance) and one output variable (Average Happiness Score). In the second stage, the ROC Analysis was conducted for the diagnostic performance of input and output variables in determining the Efficiency Status of G7 countries.

Related Works

Even though there are many studies on the subject, this one, which also uses Slack-Based Measure, has a special value because there is still a research gap regarding the effectiveness of the cohesion of education and happiness in G7 countries.

A population with higher levels of education fosters social stability, productivity and innovation (Kokkinopoulou et al., 2025), and Data Envelopment Analysis (DEA) can be considered a productivity measurement tool in a number of research areas, such as public administration, business, education, and healthcare (Charles & Kumar, 2012; Emrouznejad & Cabanda, 2014; Gregoriou & Zhu, 2005). Table 1 presents a few studies that used Data Envelopment Analysis in the fields of education, happiness, and related subjects.

Table 1. Literature Summary

Title	Author(s)	Input(s)	Output(s)	DMUs	Analysis Procedure
Managerial practices and school efficiency: a data envelopment analysis across OECD and MENA countries using TIMSS 2019 data	(Bhutoria & Aljabri, 2022)	- General resource availability - Number of computers - Annual instructional hours - Home educational resources - Share of students speaking the language of the test	- Mathematics achievement score - Science achievement score	26 OECD and MENA Countries	Non-Radial Data Envelopment Analysis and Tobit Regression
Evaluating Educational Performance of OECD Countries with Common-Weight DEA-Based Models	(Ucar & Karsak, 2023)	- Teachers/ Students Ratio - Learning Time - The Average Number of Computers Available in Schools for	- PISA Reading and Test Scores - PISA Mathematics Test Scores	37 OECD Countries	Common-Weight Data Envelopment Analysis

Title	Author(s)	Input(s)	Output(s)	DMUs	Analysis Procedure
		Educational Purposes	- PISA Science Test Scores		
A DEA-inspired model to evaluate the efficiency of education in OECD countries	(Contreras Rubio & Dominguez-Gil, 2021)	-The Cumulative Expenditure by Educational Institutions Per Student From 6 To 15 Years Old -The Student-Teacher Ratio -The Number of Computers Available for Educational Purposes -The Time Per Week Spent in School in Regular Mathematics Lessons	-The Percentages of Students That Achieve Each Proficiency Level	37 OECD Countries	Additive Data Envelopment Analysis
Education Efficiency and Labor Market Achievements: An Evaluation for Twenty OECD Countries	(Vliamos & Tzeremes, 2006)	- Survival Rates in Tertiary Education - Annual Expenditure on Educational Institutions Per Student - Public Subsidies for Households and Other Private Entities - School Expectancy for All Levels of Education combined - Ratio of Students to Teaching Staff in Educational Institutions - Number of Teaching Hours per Year	-Trends in Employment Ratios by Educational Attainment - Relative Earnings of the Population with Income from Employment by Level of Educational Attainment	20 OECD Countries	Output-Orientation CCR- Data Envelopment Analysis
Exploring factors affecting the level of happiness across countries: A conditional robust nonparametric frontier analysis	(Cordero et al., 2017)	-Health -Education -Incom	-Subjective-Well-Being	20 OECD Countries	Robust Data Envelopment Analysis

In line with Table-1, the efficiency of seven G7 countries in the context of the cohesion of education and happiness is assessed using Slack-Based Data Envelopment Analysis and the diagnostic performance of inputs and output variables in determining the efficiency status of G7 countries is tested using the ROC Analysis. The following sections provide a detailed explanation of the three input variables and one output variable used in the analysis.

Methodology

Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a nonparametric technique for evaluating and contrasting the efficiency of decision-making units (DMUs), which include businesses, governments, and nations, and share input and output factors (Ray, 2004; Tone, 2017).

Radial and nonradial models are the two types of models used in DEA. (Tone, 2017). The Constant Returns to Scale (CRS) model, also called the CCR (Charnes-Cooper-Rhodes) model, was first presented by Charnes et al. and is the most fundamental Radial DEA model in 1978 (Charnes et al., 1978). The CCR (CRS) model provides an objective evaluation of overall efficiency, pinpoints the causes, and calculates the quantity of inefficiencies (Charnes et al., 1994).

Later, Banker et al. developed Variable Returns to Scale (VRS), also called the BCC (Banker-Charnes-Cooper) model, as a representative extension of Radial DEA in 1984 (Banker et al., 1984). By estimating pure technical efficiency and determining potential increasing, decreasing, or constant returns to scale, the BCC (VRS) model differentiates between technical and scale inefficiencies (Charnes et al., 1994). Whereas CCR efficiency scores are regarded as technical efficiency, BCC efficiency scores are regarded as pure technical efficiency (Ozcan, 2014).

Output-Oriented DEA seeks to maximise output production, subject to the specified resource level, whereas Input-Oriented DEA seeks to produce the observed outputs with the fewest inputs (Ramanathan, 2003).

In both models, efficiency scores of 1.00 are considered efficient; in the input-oriented models, efficiency scores between 0.00 and 1.00 are considered inefficient; in the output-oriented models, efficiency scores greater than 1 are considered inefficient (Ozcan, 2014).

Since the Slack-Based Measure (SBM) DEA satisfies the properties Units Invariant, Monotone, and Reference-Set Dependent that the CCR and BCC models do not, it can directly address the input excesses and output shortfalls of the DMU under evaluation as a Non-Radial DEA (Tone, 2001).

CCR-Input Orientation (1), CCR-Output Orientation (2), BCC-Input Orientation (3), BCC-Output Orientation (4), SBM-CRS-Input Orientation (5), SBM-CRS-Output Orientation (6), SBM-VRS-Input Orientation (7), and SBM-VRS-Output Orientation (8) are the fractional programming formulas (Emrouznejad & Cabanda, 2014; Ozcan, 2009, 2014; Tone, 2001, 2017);

$$
\begin{aligned}
& Eff = \min_{u_r v_i} \sum_{l} V_l \, X_{1 l_0} \\
& \text{s.t.} \\
& \sum_r u_r y_{rj} - \sum_i v_i x_{ij} \leq 0 \quad ; \forall j \\
& \sum_r u_r y_{rj_0} = 1 \\
& u_r, v_i \geq 0 \quad ; \forall r, \forall i.
\end{aligned}
$$

(1)

$$
\begin{aligned}
& Eff = \max_{u_r v_i} \sum_r u_r y_{rj_0} \\
& \text{s.t.} \\
& \sum_r u_r y_{rj} - \sum_i v_i x_{ij} \leq 0 \quad ; \forall j \\
& \sum_i v_i x_{ij_0} = 1 \\
& u_r, v_i \geq 0 \quad ; \forall r, \forall i.
\end{aligned}
$$

(2)

$$\begin{aligned} & \min_{\lambda,\emptyset,s_i^-,s_r^+} \emptyset \\ & \text{s.t.} \\ & \sum_j \lambda_j x_{ij} + S_i^- = \emptyset x_{ij_o} \quad \forall i \\ & \sum_j \lambda_j y_{rj} - S_r^+ = y_{rj_o} \quad \forall r \\ & \sum_j \lambda_j = 1 \\ & s_i^-, s_r^+ \geq 0 \quad \forall i, \forall r \\ & \lambda_j \geq 0 \quad \forall_j. \end{aligned} \tag{3}$$

$$\begin{aligned} & \max_{\lambda,\emptyset,s_i^-,s_r^+} \theta \\ & \text{s.t.} \\ & \sum_j \lambda_j x_{ij} + S_i^- = x_{ij_o} \quad \forall i \\ & \sum_j \lambda_j y_{rj} - S_r^+ = \theta y_{rj_o} \quad \forall r \\ & \sum_j \lambda_j = 1 \\ & s_i^-, s_r^+ \geq 0 \quad \forall i, \forall r \\ & \lambda_j \geq 0 \quad \forall_j. \end{aligned} \tag{4}$$

$$\begin{aligned} & \rho_I^* = \min_{\lambda,s^-,s^+} 1 - \frac{1}{m}\sum\nolimits_{i=1}^{m} \frac{s_i^-}{x_{ih}} \\ & \text{subject to} \\ & x_{ih} = \sum\nolimits_{j=1}^{n} x_{ij}\lambda_j + s_i^- \quad (i = 1, \ldots, m) \\ & y_{rh} = \sum\nolimits_{j=1}^{n} y_{rj}\lambda_j - s_r^+ \quad (r = 1, \ldots, s) \\ & \lambda_j \geq 0 \ (\forall j), \ s_i^- \geq 0 (\forall i), \ s_r^+ \geq 0 (\forall r) \end{aligned} \tag{5}$$

$$\begin{aligned} & 1/\rho_O^* = \max_{\lambda,s^-,s^+} 1 + \frac{1}{s}\sum\nolimits_{r=1}^{s} \frac{s_r^+}{y_{rh}} \\ & \text{subject to} \\ & x_{ih} = \sum\nolimits_{j=1}^{n} x_{ij}\lambda_j + s_i^- \quad (i = 1, \ldots, m) \\ & y_{rh} = \sum\nolimits_{j=1}^{n} y_{rj}\lambda_j - s_r^+ \quad (r = 1, \ldots, s) \\ & \lambda_j \geq 0 \ (\forall j), \ s_i^- \geq 0 \ (\forall i), \ s_r^+ \geq 0 \ (\forall r) \end{aligned} \tag{6}$$

$$\begin{aligned} & \rho_I^* = \min_{\lambda,s^-,s^+} 1 - \frac{1}{m}\sum\nolimits_{i=1}^{m} \frac{s_i^-}{x_{ih}} \\ & \text{subject to} \\ & x_{ih} = \sum\nolimits_{j=1}^{n} x_{ij}\lambda_j + s_i^- \quad (i = 1, \ldots, m) \\ & y_{rh} = \sum\nolimits_{j=1}^{n} y_{rj}\lambda_j - s_r^+ \quad (r = 1, \ldots, s) \\ & \sum\nolimits_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0 \ (\forall j), \ s_i^- \geq 0 \ (\forall i), \ s_r^+ \geq 0 \ (\forall r) \end{aligned} \tag{7}$$

$$\begin{aligned} & \rho_I^* = \min_{\lambda,s^-,s^+} 1 - \frac{1}{m}\sum\nolimits_{i=1}^{m} \frac{s_i^-}{x_{ih}} \\ & \text{subject to} \\ & x_{ih} = \sum\nolimits_{j=1}^{n} x_{ij}\lambda_j + s_i^- \quad (i = 1, \ldots, m) \\ & y_{rh} = \sum\nolimits_{j=1}^{n} y_{rj}\lambda_j - s_r^+ \quad (r = 1, \ldots, s) \\ & \sum\nolimits_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0 \ (\forall j), \ s_i^- \geq 0 \ (\forall i), \ s_r^+ \geq 0 \ (\forall r) \end{aligned} \tag{8}$$

ROC Analysis

In the past, the ROC analysis was used to help radar operators determine whether a blip on the radar screen indicated a moving object or a sound during World War II. Later, diagnostic statistical research adopted this technique (Roumeliotis et al., 2024). The ROC curve is a basic tool for examining the trade-off between sensitivity and specificity across different thresholds for classifier decisions (Tec, 2025). Using a receiver operating characteristic (ROC) graph, classifiers can be arranged, visualised, and chosen according to their performance (Fawcett, 2006).

Table 2 displays the classification table of AUC values and their usability. The Area Under the Curve (AUC) value is a commonly used metric that summarises the diagnostic performance of an index test (Çorbacıoğlu & Aksel, 2023).

Table 2. Area Under the Curve Values and its Interpretation (Çorbacıoğlu & Aksel, 2023).

AUC Value	Interpretation Suggestion
0.9 ≤ AUC	Excellent
0.8 ≤ AUC < 0.9	Considerable
0.7 ≤ AUC < 0.8	Fair
0.6 ≤ AUC < 0.7	Poor
0.5 ≤ AUC < 0.6	Fail

The other widely used scale is the Confidence Interval (CI) of 0 to 100 per cent, where a CI of 0 indicates that the observer is certain that the disease of interest is not present, and a CI of 100 indicates that the observer is certain that the disease of interest is present (Obuchowski, 2005). Because sample data are subject to statistical errors and are not fixed values, the AUC is frequently presented with a 95 per cent confidence interval (CI), which offers a range of potential values around the actual value (Nahm, 2022).

While the specificity, also known as the true negative fraction (TNF), describes the likelihood of a negative test result in non-diseased individuals, the sensitivity, also known as the true positive fraction (TPF), describes the fraction of diseased patients who actually have a positive test result. The resulting sensitivity and specificity are both 100% (Van Erkel & Pattynama, 1998).

Because using less strict criteria to increase sensitivity results in a trade-off where specificity decreases, it is imperative to set a cut-off value with an appropriate sensitivity and specificity (Nahm, 2022). ROC analysis makes it possible to assess various cut-off points and determine each one's sensitivity and specificity (He et al., 2025).

The Youden index, which is computed as sensitivity + specificity – 1, helps choose a threshold where both metrics reach their peak by determining the threshold value that maximises both sensitivity and specificity (Çorbacıoğlu & Aksel, 2023). The distance between the 45° diagonal and the ROC curve when moving the 45° diagonal—a straight line with a slope of 1—in the (0, 1) direction is known as Youden's J statistic (Nahm, 2022). The Youden index test, which finds the threshold value corresponding to the point of the curve closest to the upper left corner (corresponding to 100% specificity and 100% sensitivity), was used to determine the optimal cut-off value (Roumeliotis et al., 2024).

The Sample and Dataset

Decision Making Units (DMUs) are such units that utilize same inputs to produce the same outputs (Cooper et al., 2006). The sample of the study consists of G7 countries (Canada, France, Germany, Italy, Japan, theUnited Kingdom and the United States) and is shown in Table-1.

Since the sample of the study consists of all G7 countries, the following three conditions related to the homogeneous group of DMUs are met (Golany & Roll, 1989);

- The DMUs in question carry out identical tasks with comparable goals,
- Every DMU operates in the same market environment,
- Every DMU in the group uses the same inputs and outputs to describe its performance.

The method used to specify the feasible set of input-output bundles determines the efficiency assessment (Ray, 2004).

There are three input variables and one output variable as the primary input/output framework. Inputs are i1: PISA Reading Performance, i2: PISA Mathematics Performance and i3: PISA Science Performance. Output is o1: Average Happiness Score. The inputs and outputs are in accordance with (Bhutoria & Aljabri, 2022; Contreras Rubio & Dominguez-Gil, 2021;

Cordero et al., 2017; Ucar & Karsak, 2023; Vliamos & Tzeremes, 2006), as presented in Table 2.

The dataset, which belongs to 2025 or the nearest year, is shown in Table 3 and was gathered from (Helliwell et al., 2024; OECD, 2023).

It is anticipated that the number of DMUs will exceed the sum of the input and output numbers (Ramanathan, 2003). This requirement is satisfied because the study uses four variables in total—three input variables and one output variable—to assess the effectiveness of the seven G7 nations.

Table 3. The Dataset

DMU Name	Inputs			Output
	i1	i2	i3	o1
Canada	507.132896	496.947894	515.016676	6.803000
France	473.852158	473.944418	487.225279	6.593000
Germany	479.794025	474.826454	492.426730	6.753000
Italy	481.598278	471.259476	477.463366	6.415000
Japan	515.854614	535.579306	546.634454	6.147000
United Kingdom	494.404626	488.975085	499.670214	6.728000
United States	503.937574	464.888043	499.414064	6.724000
Min	515.854614	535.579306	546.634454	6.803000
Max	473.852158	464.888043	477.463366	6.147000
Mean	493.796310	486.631525	502.550112	6.594714
Median	494.404626	474.826454	499.414064	6.724000

According to Table 3;

- ***i1: PISA Reading Performance***, for PISA, measures the capacity of a 15-year-old to understand, use and reflect on written texts to achieve personal goals, develop knowledge and potential, and participate in society, and relevant data is gathered from OECD Data (OECD, 2023).
- ***i2: PISA Mathematics Performance***, as assessed by PISA, measures the ability of 15-year-olds to use mathematics to solve real-world problems, and relevant data is gathered from OECD Data (OECD, 2023).
- ***i3: PISA Science Performance***, for PISA, measures the scientific literacy of a 15-year-old in using scientific knowledge to identify questions, acquire new knowledge, explain scientific phenomena, and draw evidence-based conclusions about science-related issues, and relevant data is gathered from OECD Data (OECD, 2023).
- ***o1: Average Happiness Score*** is based on a single question that asks people to evaluate the quality of their life on a 0–10 scale, and relevant data is gathered from the World Happiness Report 2025 (Helliwell et al., 2024).

Analysis Design

The study has a two-stage analysis design. In the first stage; Input Orientation Slack-Based Data Envelopment Analysis (DEA) model was employed using five input variables (PISA Reading Performance, PISA Mathematics Performance and PISA Science Performance) and one output variable (Average Happiness Score). The diagnostic performance of input and output variables in assessing the G7 countries' efficiency status was assessed in the second stage using ROC analysis.

During the DEA Stage;

- Since the managers of the G7 countries have more control over inputs than outputs (Ozcan, 2014), Slack-Based Measure deals directly with input excess and output shortfall (Tone, 2001). The Input-Oriented Slack-Based Data Envelopment Analysis (DEA) model was employed in this research.
- In order to make inefficient DMUs more efficient, reference sets were found for each one, and options for improvement (lowering inputs or raising outputs) were computed.
- DEA was performed using the deaR package in the R project (Coll-Serrano et al., 2018).

During the ROC Analysis Stage;

- ROC analysis was used for evaluating for evaluating the accuracy of the DEA, which classifies G7 countries into 1 of 2 categories; efficient or inefficient (Zou et al., 2007).
- The AUC value was used for evaluating the test’s ability to distinguish between efficient and inefficient G7 countries (Çorbacıoğlu & Aksel, 2023).
- For any test to be statistically significant, the lower 95% CI value of the AUC must be >0.5 (Nahm, 2022).
- The optimal cut-off value for the test that maximises sensitivity and specificity was identified (Roumeliotis et al., 2024).
- The Youden index was used to identify the optimal cutoff value (Çorbacıoğlu & Aksel, 2023).
- ROC Analysis was performed using the Inonu University Faculty of Medicine, Department of Biostatistics and Medical Informatics, Diagnostic Tests and ROC Analysis Software (Yaşar et al., 2025).

Results

The Results of the DEA Stage

Table 4 displays the efficiency scores, slacks (input excesses and output shortfalls), and the G7 countries' (DMUs') ranking based on their efficiency scores.

Table 4. Efficiency Score, Ranking, and Slacks of DMUs

DMU	Efficiency Score	Efficiency Status	Rank	s_1^-	s_2^-	s_3^-	s_1^+
Canada	1.000000	Efficient	1	507.132896	496.947894	515.016676	6.803000
France	1.000000	Efficient	1	473.852158	473.944418	487.225279	6.593000
Germany	1.000000	Efficient	1	479.794025	474.826454	492.426730	6.753000
Italy	1.000000	Efficient	1	481.598278	471.259476	477.463366	6.415000
Japan	0.895653	Inefficient	7	481.598278	471.259476	477.463366	6.415000
United Kingdom	0.974410	Inefficient	6	478.865608	474.688636	491.614003	6.728000
United States	1.000000	Efficient	1	503.937574	464.888043	499.414064	6.724000

According to Table 4, five G7 countries (Canada, France, Germany, Italy and the United States) have a score of 1, and they are relatively efficient by removing input excesses and augmenting the output shortfalls. Besides, all five efficient G7 countries ranked first, while Japan ranked last among the seven G7 countries.

Furthermore, Table 4 gives the input and output slacks so that each of the G7 nations can eliminate excess inputs and make up for output deficiencies in order to reach efficiency status.

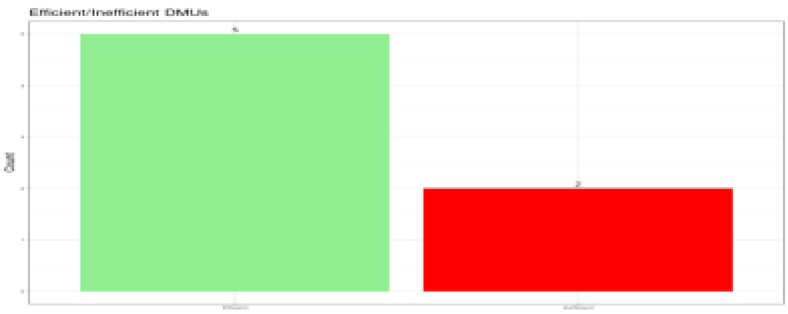

Figure 1. Efficiency Distribution

Five of the seven G7 nations are efficient (yellow column), and two are inefficient (red column), as shown in Figure 1.

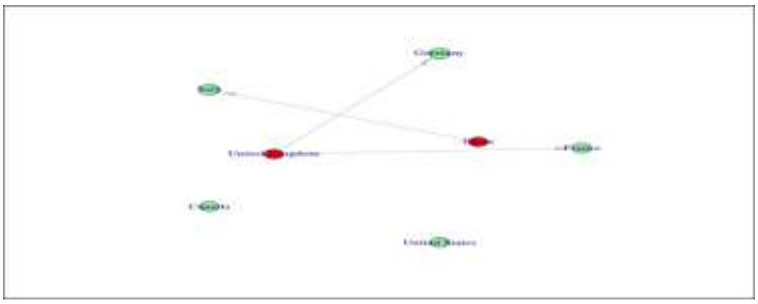

Figure 2. Network Graph

Figure 2 shows Reference Sets defined so that the two inefficient G7 regions can become efficient. The green circles in Figure 2 represent the efficient G7 countries, while the red circles represent the inefficient ones.

Table 5 demonstrates the reference set weights, also known as Lambda -λ- values, which define the components of other producers used to construct the virtual producer (Anderson, 2003), of the Reference Set so that the two inefficient G7 countries can become efficient.

Table 5. References Set of Inefficient States (λ)

DMU	France (λ)	Germany (λ)	Italy (λ)
Japan	0.0000	0.0000	1.0000
United Kingdom	0.1562	0.8438	0.0000

According to Table 5;

- The reference set of Japan consists of Italy (λ=1.0000).
- The reference set of the United Kingdom consists of France (λ=0.1562) and Germany (λ=0.8438).

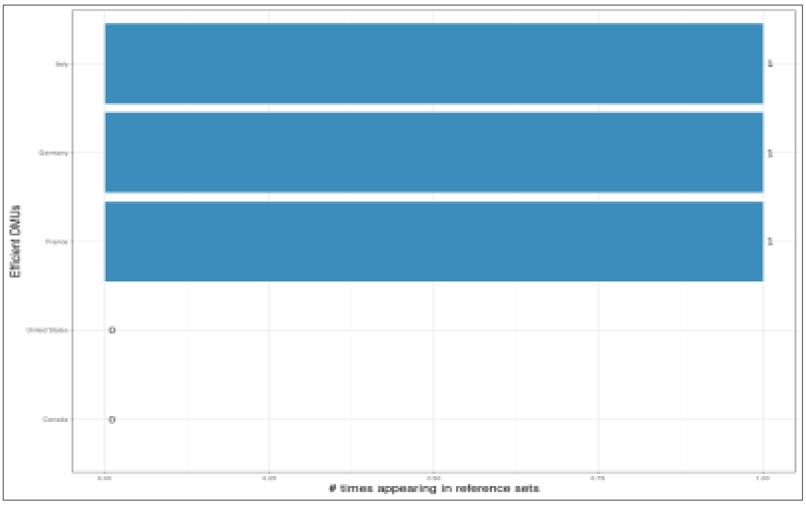

Figure 3. Reference (Peer) Counts

According to Figure 3, Italy, Germany and France are the most appeared G7 countries in the reference sets with once each peer counts. Furthermore, although the United States and Canada are efficient, they have not appeared in the reference sets.

Table 6 lists improvement options (increasing outputs or decreasing inputs) for inefficient G7 nations.

Table 6. Improvement Options for Inefficient Decision-Making Units (DMUs)

DMU	i^*1	i^*2	i^*3	o^*1
Japan	-34.2563	-64.3198	-69.1711	0.2680
United Kingdom	-15.5390	-14.2864	-8.0562	0.0000

According to Table 5;

- ***Japan***, with i*1 *(-34.2563)* value, is the G7 country that needs the most improvement in terms of ***i1: PISA Reading Performance*** input variable.
- ***Japan***, with i*2 *(-64.3198)* value, is the G7 country that needs the most improvement in terms of ***i2: PISA Mathematics Performance*** input variable.
- ***Japan***, with i*3 *(-0.691711)* value, G7 country that needs the most improvement in terms of ***i3: PISA Science Performance*** input variable.
- ***Japan***, with o*1 *(0.2680) value*, G7 country that needs the most improvement in terms of ***o1: Average Happiness Score*** output variable.

The Results of the ROC Analysis Stage

The result of the ROC Analysis conducted for the diagnostic performance of input and output variables in determining the Efficiency Status of G7 countries is shown in Table 7.

Table 7. Diagnostic Performance in Determining the Efficiency Status of Input and Output Variables

Variables	Groups	Median (IQR)	AUC (%95 CI)	Z- Test	p	Youden Index	Cut- off Value	Sensitivity (%)	Specificity (%)
i1: PISA Reading Performance	Inefficient	481.5983 (24.1435)	0.8 (0.34-1)	1.279	0.201	0.6	488.001	1	0.6
	Efficient	505.1296 (10.725)							
i2: PISA Mathematics Performance	Inefficient	473.9444 (3.567)	0.9 (0.623-1)	2.828	0.005	0.8	481.901	1	0.8
	Efficient	512.2772 (23.3021)							
i3: PISA Science Performance	Inefficient	492.4267 (12.1888)	0.9 (0.623-1)	2.828	0.005	0.8	499.542	1	0.8
	Efficient	523.1523 (23.4821)							
o1: Average Happiness Score	Inefficient	6.4375 (0.2905)	0.7 (0.065-1)	0.617	0.537	0.5	6.281	0.5	1
	Efficient	6.724 (0.16)							

AUC; Area Under Curve, CI; Confidence Interval

It was determined that the *i2: PISA Mathematics Performance* input variable *(AUC=0.9 [95%CI: 0.623-1]; p=0.005; Figure 4.)* could distinguish the Efficiency Status. In this input variable, sensitivity *(1)* and specificity *(0.8)* were calculated for the cutoff point *(481.901)* determined by the Youden index *(0.8)*.

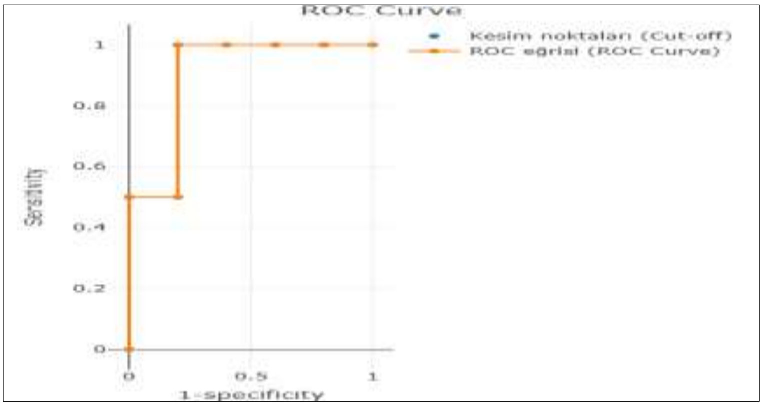

Figure 4. ROC Curve in Determining the Efficiency Status (for i2: PISA Mathematics Performance Input Variable)

It was determined that the i3: PISA Science Performance input variable (AUC=0.9 [95%CI: 0.623-1]; p=0.005; Figure 5) could distinguish the Efficiency Status. In this input variable, sensitivity (1) and specificity (0.8) were calculated for the cutoff point (499.542) determined by the Youden index (0.8).

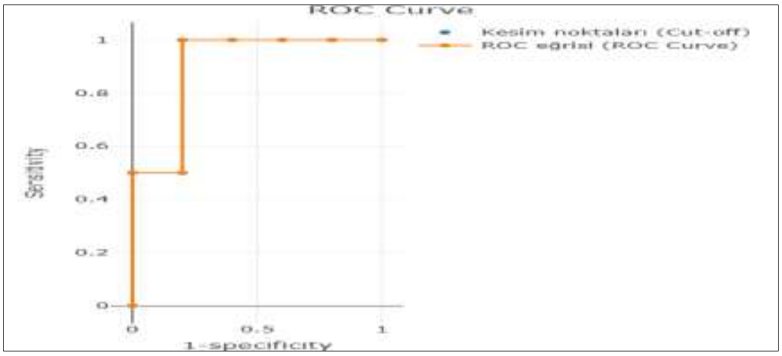

Figure 5. ROC Curve in Determining the Efficiency Status (for i3: PISA Science Performance Input Variable)

Sensitivity (1) and specificity (0.6) were found for the cutoff point (488.001) determined by the Youden index (0.6) in the i1: PISA Reading Performance input variable. However, this input variable was not identified as the diagnostic factor in determining Efficiency Status (p=0.201; Figure 6).

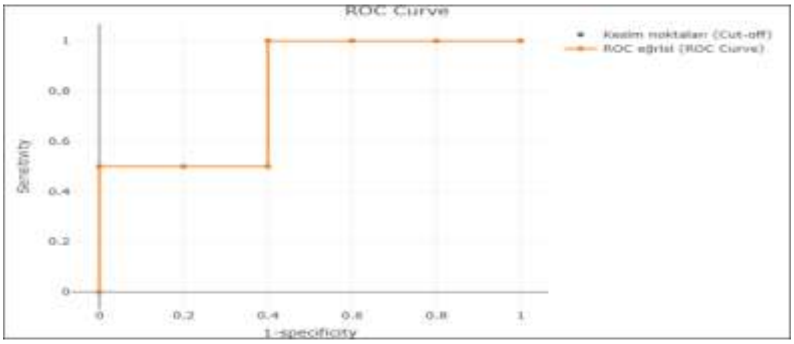

Figure 6. ROC Curve in Determining the Efficiency Status (for i1: PISA Reading Performance Input Variable)

Sensitivity (0.5) and specificity (1) were found for the cutoff point (6.281) determined by the Youden index (0.5) in the o1: Average Happiness Score output variable. However, this output variable was not identified as the diagnostic factor in determining Efficiency Status (p=0.537; Figure 7).

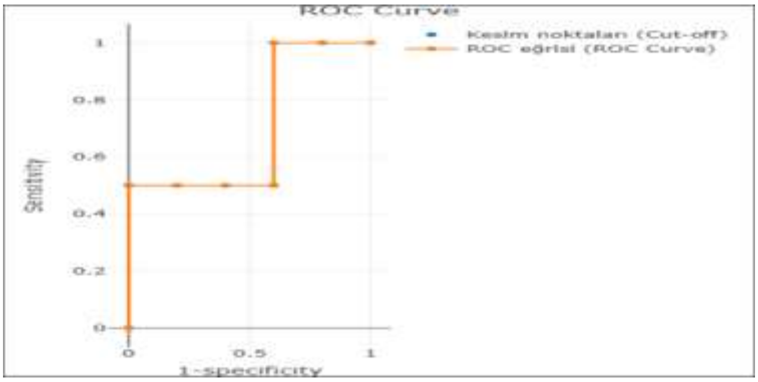

Figure 7. ROC Curve in Determining the Efficiency Status (for o1: Average Happiness Score Output Variable)

Discussion and Conclusion

The study has two main objectives: (i) to assess the efficiency of the cohesion of education and happiness of G7 countries *(Canada, France, Germany, Italy, Japan, the United Kingdom and the United States)*, (ii) to test the diagnostic performance of input and output variables in determining the efficiency status of G7 countries.

Therefore, the study has a two-stage analysis design. In the first stage; Input Orientation Slack-Based Data Envelopment Analysis (DEA) model was employed using five input variables (PISA Reading Performance, PISA Mathematics Performance and PISA Science Performance) and one output variable (Average Happiness Score). In the second stage, the ROC Analysis was conducted for the diagnostic performance of input and output variables in determining the Efficiency Status of G7 countries.

According to the first stage of the analysis design;

- Five G7 countries (Canada, France, Germany, Italy and the United States) have a score of 1, and they are relatively efficient. This finding is in line with (Vliamos & Tzeremes, 2006), in terms of Italy. However, two G7 countries (Japan and the United Kingdom) are inefficient. This finding is not in line with (Bhutoria & Aljabri, 2022). It is believed that the input and output bundles and research questions of this study are the cause of this circumstance. Also, this finding is in line with (Ucar & Karsak, 2023). Moreover, this finding is in line with (Contreras Rubio & Dominguez-Gil, 2021), in terms of the United Kingdom.
- All five efficient G7 countries rank first, while Japan ranks last among the seven G7 countries. This finding is not in line with (Bhutoria & Aljabri, 2022). Japan has the lowest Average Happiness Score in our dataset, which is assumed to be the cause of this circumstance. Also, this finding is in line with (Cordero et al., 2017; Ucar & Karsak, 2023).

- Italy, Germany and France are the most appeared G7 countries in the reference sets with once each peer counts. This finding is in line with (Vliamos & Tzeremes, 2006), in terms of Italy.
- Although the United States and Canada are efficient, they have not appeared in the reference sets. Unlike other efficient G7 countries, it is believed that this situation is a result of the United States and Canada being situated in the Americas.
- When improvement options (reducing of the inputs or increasing of the outputs) for inefficient G7 countries are examined;
  - ✓ Japan, with an i*1 (-34.2563) value, is the G7 country that needs the most improvement in terms of i1: PISA Reading Performance input variable Japan has the highest i1: PISA Reading Performance value in our dataset, which is thought to be the cause of this situation.
  - ✓ Japan, with an i*2 (-64.3198) value, is the G7 country that needs the most improvement in terms of i2: PISA Mathematics Performance input variable. Japan's highest i2: PISA Mathematics Performance value in our dataset is thought to be the cause of this circumstance.
  - ✓ Japan, with an i*3 (-0.691711) value, G7 country that needs the most improvement in terms of i3: PISA Science Performance input variable. According to our dataset, Japan has the highest i3: PISA Science Performance value, which is believed to be the cause of this situation.
  - ✓ Japan, with an o*1 (0.2680) value, G7 country that needs the most improvement in terms of o1: Average Happiness Score output variable. This is believed to be the result of Japan having the lowest o1: Average Happiness Score value in our dataset.

According to the second stage of the analysis design;

- It was determined that the i2: PISA Mathematics Performance and i3: PISA Science Performance input variables could distinguish the Efficiency Status with the cutoff points (481.901) and (499.542), respectively.
- i1: PISA Reading Performance input variable and o1: Average Happiness Score output variable were not identified as the diagnostic factor in determining Efficiency Status.

In order to implement more effective social policies, the institutions and organisations of the G7 countries that deal with education management are expected to benefit somewhat from the findings of this study. The most significant of these benefits is believed to be the chance for inefficient G7 nations to improve their social policies and management of education, including the financial and non-financial costs in terms of national accounts, by closely examining, contrasting, and modelling the practices of G7 nations in the reference sets established for them. Furthermore, the relevant organisations of ineffective G7 nations need to know the cut-off values for i2: PISA Mathematics Performance and i3: PISA Science Performance to restructure their educational systems

Therefore, using various input/output bundles on the larger samples, researchers in the areas of education management and social policies are advised to conduct performance evaluation using Integrated Two-Stage Data Envelopment Analysis and ROC Analysis

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